OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 4

4731

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 1 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A wheel is rotating and is slowing down with constant angular deceleration. The initial angular speed is $80\,\mathrm{rad\,s^{-1}}$, and after 15 s the wheel has turned through 1020 radians.

(i) Find the angular deceleration of the wheel. **[2]**

(ii) Find the angle through which the wheel turns in the last 5 s before it comes to rest. **[2]**

(iii) Find the total number of revolutions made by the wheel from the start until it comes to rest. **[3]**

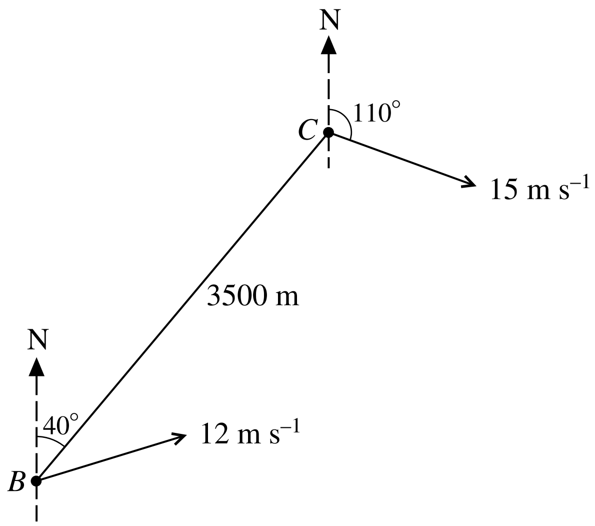
2 The region bounded by the x-axis, the y-axis, the line $x = \ln 3$, and the curve $y = \mathrm{e}^{-x}$ for $0 \leqslant x \leqslant \ln 3$, is occupied by a uniform lamina. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

3 A circular disc is rotating in a horizontal plane with angular speed $16\,\mathrm{rad\,s^{-1}}$ about a fixed vertical axis passing through its centre O. The moment of inertia of the disc about the axis is $0.9\,\mathrm{kg\,m^2}$. A particle, initially at rest just above the surface of the disc, drops onto the disc and sticks to it at a point 0.4 m from O. Afterwards, the angular speed of the disc with the particle attached is $15\,\mathrm{rad\,s^{-1}}$.

(i) Find the mass of the particle. **[4]**

(ii) Find the loss of kinetic energy. **[3]**

4



From a boat B, a cruiser C is observed 3500 m away on a bearing of 040°. The cruiser C is travelling with constant speed $15\,\mathrm{m\,s^{-1}}$ along a straight line course with bearing 110° (see diagram). The boat B travels with constant speed $12\,\mathrm{m\,s^{-1}}$ on a straight line course which takes it as close as possible to the cruiser C.

(i) Show that the bearing of the course of B is 073°, correct to the nearest degree. **[4]**

(ii) Find the magnitude and the bearing of the velocity of C relative to B. **[3]**

(iii) Find the shortest distance between B and C in the subsequent motion. **[3]**

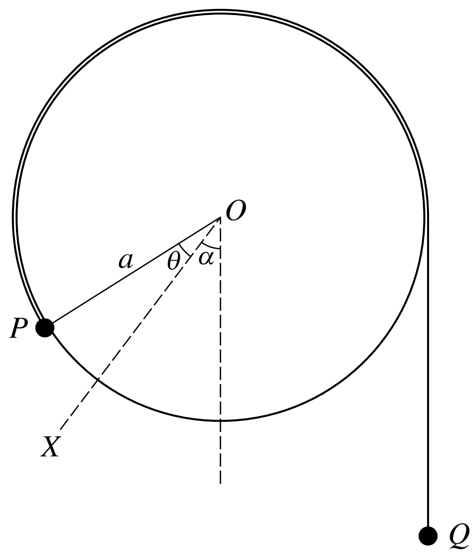
5 A uniform rod AB has mass m and length $6a$. The point C on the rod is such that $AC = a$. The rod can rotate freely in a vertical plane about a fixed horizontal axis passing through C and perpendicular to the rod.

(i) Show by integration that the moment of inertia of the rod about this axis is $7ma^2$. **[5]**

The rod starts at rest with B vertically below C. A couple of constant moment $\dfrac{6mga}{\pi}$ is then applied to the rod.

(ii) Find, in terms of a and g, the angular speed of the rod when it has turned through one and a half revolutions. **[6]**

6



A light pulley of radius a is free to rotate in a vertical plane about a fixed horizontal axis passing through its centre O. Two particles, P of mass $5m$ and Q of mass $3m$, are connected by a light inextensible string. The particle P is attached to the circumference of the pulley, the string passes over the top of the pulley, and Q hangs below the pulley on the opposite side to P. The section of string not in contact with the pulley is vertical. The fixed line OX makes an angle α with the downward vertical, where $\cos\alpha = \frac{4}{5}$, and OP makes an angle θ with OX (see diagram).

You are given that the total potential energy of the system (using a suitable reference level) is V, where

$$V = mga(3\sin\theta - 4\cos\theta - 3\theta).$$

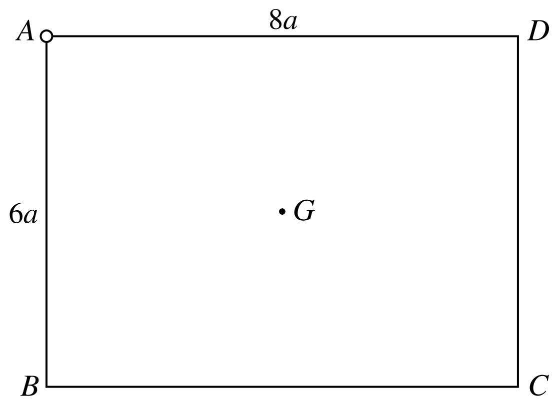
(i) Show that $\theta = 0$ is a position of stable equilibrium. **[5]**

(ii) Show that the kinetic energy of the system is $4ma^2\dot{\theta}^2$. **[2]**

(iii) By differentiating the energy equation, then making suitable approximations for $\sin\theta$ and $\cos\theta$, find the approximate period of small oscillations about the equilibrium position $\theta = 0$. **[5]**

[Question 7 is printed overleaf.]

7



The diagram shows a uniform rectangular lamina $ABCD$ with $AB = 6a$, $AD = 8a$ and centre G. The mass of the lamina is m. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A and perpendicular to the lamina.

(i) Find the moment of inertia of the lamina about this axis. **[3]**

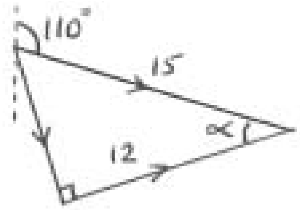
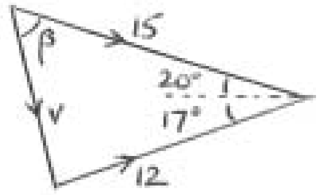
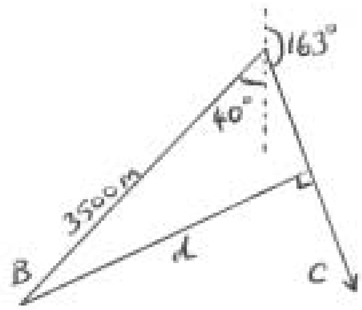
The lamina is released from rest with AD horizontal and BC below AD.

(ii) For an instant during the subsequent motion when AD is vertical, show that the angular speed of the lamina is $\sqrt{\frac{3g}{50a}}$ and find its angular acceleration. **[5]**

At an instant when AD is vertical, the force acting on the lamina at A has magnitude F.

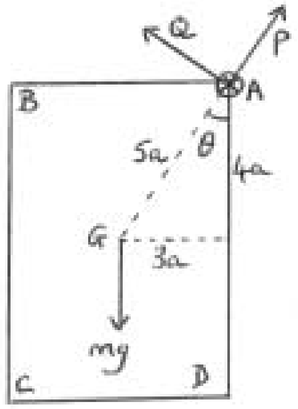
(iii) By finding components parallel and perpendicular to GA, or otherwise, show that $F = \frac{\sqrt{493}}{20}mg$. **[8]**

1 (i)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $1020 = 80\times15 + \frac{1}{2}\alpha\times15^2$ $\alpha = -1.6$ Angular deceleration is $1.6\ \text{rad s}^{-2}$	M1 A1 **[2]**	
(ii)	Using $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$, $\theta = 0 - \frac{1}{2}\times(-1.6)\times5^2$ Angle is 20 rad	M1 A1 ft **[2]**	ft is $12.5\lvert\alpha\rvert$
(iii)	Using $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$, $0 = 80^2 + 2\times(-1.6)\theta$ $\theta = 2000$ Number of revolutions is 318 (3 sf)	M1 A1 ft A1 **[3]**	*Accept* $\dfrac{1000}{\pi}$
2	Area is $\int_0^{\ln 3} e^{-x}\,dx$ $= \left[-e^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3})$ $\int xy\,dx = \int_0^{\ln 3} x e^{-x}\,dx$ $= \left[-xe^{-x} - e^{-x} \right]_0^{\ln 3} \quad (= \frac{2}{3} - \frac{1}{3}\ln 3)$ $\bar{x} = \dfrac{\frac{2}{3} - \frac{1}{3}\ln 3}{\frac{2}{3}} = 1 - \frac{1}{2}\ln 3$ $\int \frac{1}{2}y^2\,dx = \int_0^{\ln 3} \frac{1}{2}(e^{-x})^2\,dx$ $= \left[-\frac{1}{4}e^{-2x} \right]_0^{\ln 3} \quad (= \frac{2}{9})$ $\bar{y} = \dfrac{\frac{2}{9}}{\frac{2}{3}} = \dfrac{1}{3}$	M1 A1 M1 M1 A1 A1 M1 A1 A1 **[9]**	*Limits not required* For $-e^{-x}$ *Limits not required* Integration by parts For $-xe^{-x} - e^{-x}$ $\int (e^{-x})^2\,dx$ or $\int(-\ln y)y\,dy + (\frac{1}{3}\ln 3)\times\frac{1}{6}$ $-\frac{1}{4}e^{-2x}$ or $-\frac{1}{2}y^2\ln y + \frac{1}{4}y^2$ *(dep on M1)* *Max penalty of 1 mark for correct answers in an unacceptable form (eg decimals)*
3 (i)	By conservation of angular momentum $I_2 \times 15 = 0.9\times16$ $I_2 = 0.96$ $I_2 = 0.9 + m\times0.4^2$ Mass is 0.375 kg	M1 A1 M1 A1 **[4]**	Using $I\omega$
(ii)	KE before is $\frac{1}{2}\times0.9\times16^2$ KE after is $\frac{1}{2}\times0.96\times15^2$ Loss of KE is $115.2 - 108 = 7.2\ \text{J}$	M1 A1 ft A1 **[3]**	Using $\frac{1}{2}I\omega^2$ Both expressions correct

4 (i)	 $\cos\alpha = \frac{12}{15}$ $\alpha = 36.87°$ (4 sf) Bearing of $\mathbf{v}_B$ is $110 - 36.87 = 073.13$ $= 073°$ (nearest degree)	M1 A1 M1 A1 ag [**4**]	Velocity triangle with $90°$ opposite $\mathbf{v}_C$ Correct velocity triangle Finding a relevant angle
(ii)	Magnitude is $\sqrt{15^2 - 12^2} = 9\ \mathrm{m\,s^{-1}}$ Direction is $90°$ from $\mathbf{v}_B$ Bearing is $73.13 + 90 = 163°$ (nearest degree)	B1 M1 A1 [**3**]	Accept 8.95 to 9.05
	Alternative for **(ii)** (*using given answer in* **(i)**)  $v^2 = 12^2 + 15^2 - 2\times12\times15\cos37°$ $v = 9$ $\frac{\sin\beta}{12} = \frac{\sin37°}{v}$ $\beta = 53°$ Bearing is $110 + 53 = 163°$	B1 M1 A1	or Relative velocity is $\begin{pmatrix} v\sin\theta \\ v\cos\theta \end{pmatrix} = \begin{pmatrix} 15\sin110 \\ 15\cos110 \end{pmatrix} - \begin{pmatrix} 12\sin73 \\ 12\cos73 \end{pmatrix} \approx \begin{pmatrix} 2.6 \\ -8.6 \end{pmatrix}$ or $v^2 = (2.6...)^2 + (-8.6...)^2$ Accept 8.95 to 9.05 Finding a relevant angle or $\tan\theta = \frac{2.6...}{-8.6...}$
(iii)	As viewed from *B*  $d = 3500\sin56.87°$ Shortest distance is 2930 m (3 sf)	M1 M1 A1 [**3**]	Diagram indicating initial displacement and relative velocity *May be implied* Accept 2910 to 2950
	Alternative for **(iii)** $d^2 = (3500\sin40° + 2.6...t)^2$ $+(3500\cos40° - 8.6...t)^2$ Minimum when $-34432 + 162t = 0$ $t = 213$ Shortest distance is 2930 m (3 sf)	M1 M1 A1	Differentiating or completing the square Accept 2910 to 2950

5 (i)	$I = \int_{-a}^{5a} \frac{m}{6a} x^2 \, dx$ or $\int_{-a}^{5a} \rho x^2 \, dx$	M1 M1 A1	$(\delta m)x^2$ or $(\rho \delta x)x^2$ or integrating x^2 Using $\delta m = \frac{m \delta x}{6a}$ or $\rho = \frac{m}{6a}$ Correct integral expression for *I* eg $I = \int_0^{5a} ... + \int_0^{a} ...$ $I = \int_{-3a}^{3a} ... + m(2a)^2$, $I = 2\int_0^{3a} ... + m(2a)^2$ $I = \int_0^{6a} ... - m(3a)^2 + m(2a)^2$
	$= \left[\frac{m}{18a} x^3 \right]_{-a}^{5a} = \frac{m}{18a}(125a^3 + a^3)$ or $42\rho a^3$ $= \frac{126ma^3}{18a} = 7ma^2$	M1 A1 ag [**5**]	Evaluating definite integral *Dependent on integrating* x^2
(ii)	WD by couple is $\frac{6mga}{\pi} \times 3\pi \quad (= 18mga)$ Gain of PE is $mg(4a)$ $18mga = 4mga + \frac{1}{2}(7ma^2)\omega^2$ Angular speed is $\sqrt{\frac{4g}{a}}$	M1 A1 B1 M1 A1 ft A1 [**6**]	Using $C\theta$ Equation involving WD, PE and $\frac{1}{2} I \omega^2$

6 (i)	$\frac{dV}{d\theta} = mga(3\cos\theta + 4\sin\theta - 3)$ When $\theta = 0$, $\frac{dV}{d\theta} = mga(3+0-3) = 0$ so $\theta = 0$ is a position of equilibrium $\frac{d^2V}{d\theta^2} = mga(-3\sin\theta + 4\cos\theta)$ When $\theta = 0$, $\frac{d^2V}{d\theta^2} = 4mga > 0$ hence the equilibrium is stable	B1 M1 A1 ag M1 A1 ag **[5]**	Considering $\frac{dV}{d\theta} = 0$ Correctly shown Considering $\frac{d^2V}{d\theta^2}$ (or other method) $V'' = 4mga \Rightarrow \text{Stable}$ M1A0 $V'' = 4mga \Rightarrow \text{Minimum} \Rightarrow \text{Stable}$ M1A1
(ii)	Speed of *P* and *Q* is $a\dot{\theta}$ KE is $\frac{1}{2}(5m)(a\dot{\theta})^2 + \frac{1}{2}(3m)(a\dot{\theta})^2$ or $\frac{1}{2}(8m)(a\dot{\theta})^2$ $= \frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2$ $= 4ma^2\dot{\theta}^2$	M1 A1 ag **[2]**	Or moment of inertia of *P* is $5ma^2$ $\frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2$ M1A1 $\frac{1}{2}(5ma^2)\dot{\theta}^2 + \frac{1}{2}(3ma^2)\dot{\theta}^2$ M1A0 $\frac{1}{2}(8ma^2)\dot{\theta}^2$ M1A0
(iii)	$V + 4ma^2\dot{\theta}^2 = K$ $\frac{dV}{d\theta}\dot{\theta} + 8ma^2\dot{\theta}\ddot{\theta} = 0$ $mga(3\cos\theta + 4\sin\theta - 3)\dot{\theta} + 8ma^2\dot{\theta}\ddot{\theta} = 0$ For small θ, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ $mga(3 + 4\theta - 3) + 8ma^2\ddot{\theta} \approx 0$ $\ddot{\theta} \approx -\frac{g}{2a}\theta$ Approximate period is $2\pi\sqrt{\frac{2a}{g}}$	M1 A1 M1 A1 ft A1 **[5]**	$= 0$ is required for A1 *(may be implied by later work)* Linear approximation (ft is dep on M1M1)

7 (i)	$I = \frac{1}{3}m\{(3a)^2 + (4a)^2\} + m(5a)^2$ $= \dfrac{100ma^2}{3}$	M1 A1 A1 [**3**]	Using parallel (or perpendicular) axes rule or $I = \frac{4}{3}m(3a)^2 + \frac{4}{3}m(4a)^2$
(ii)	 By conservation of energy, $\frac{1}{2}(\frac{100}{3}ma^2)\omega^2 = mg(4a - 3a)$ $\frac{50}{3}ma^2\omega^2 = mga$ Angular speed is $\sqrt{\dfrac{3g}{50a}}$ $-mg(3a) = (\frac{100}{3}ma^2)\alpha$ Angular acceleration is $(-)\dfrac{9g}{100a}$	M1 A1 ft A1 ag M1 A1 [**5**]	Equation involving KE and PE Using $C = I\alpha$
(iii)	$P - mg\cos\theta = m(5a)\omega^2$ $P - \frac{4}{5}mg = m(5a)\left(\dfrac{3g}{50a}\right)$ $P = \frac{11}{10}mg$ $Q - mg\sin\theta = m(5a)\alpha$ $Q - \frac{3}{5}mg = -m(5a)\left(\dfrac{9g}{100a}\right)$ $Q = \frac{3}{20}mg$ $F = \sqrt{P^2 + Q^2} = \frac{1}{20}mg\sqrt{22^2 + 3^2}$ $= \dfrac{\sqrt{493}}{20}mg$	M1 A2 M1 A2 ft M1 A1 ag [**8**]	Equation involving *P* and $r\omega^2$ Give A1 if correct apart from sign(s) *(Allow $\frac{3}{5}H + \frac{4}{5}V$ in place of P)* Equation involving *Q* and $r\alpha$ Give A1 if correct apart from sign(s) ft for wrong value of α ft for wrong value of *r* in second equation *(Allow $\frac{3}{5}V - \frac{4}{5}H$ in place of Q)* *Dependent on previous M1M1*
	Alternative for **(iii)** $H = m(5a)\omega^2\sin\theta - m(5a)\alpha\cos\theta$ $H = m(5a)\left(\frac{3g}{50a}\right)\left(\frac{3}{5}\right) + m(5a)\left(\frac{9g}{100a}\right)\left(\frac{4}{5}\right)$ $V - mg = m(5a)\omega^2\cos\theta + m(5a)\alpha\sin\theta$ $V - mg = m(5a)\left(\frac{3g}{50a}\right)\left(\frac{4}{5}\right) - m(5a)\left(\frac{9g}{100a}\right)\left(\frac{3}{5}\right)$ $H = \frac{27}{50}mg,\ \ V = \frac{97}{100}mg$	M1 A2 ft M1 A2 ft	Equation involving *H*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s) Equation involving *V*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s)

	$F=\sqrt{H^2+V^2}=\frac{1}{100}mg\sqrt{54^2+97^2}$ $=\frac{\sqrt{12325}}{100}mg=\frac{\sqrt{493}}{20}mg$	M1 A1 ag	*Dependent on previous M1M1*